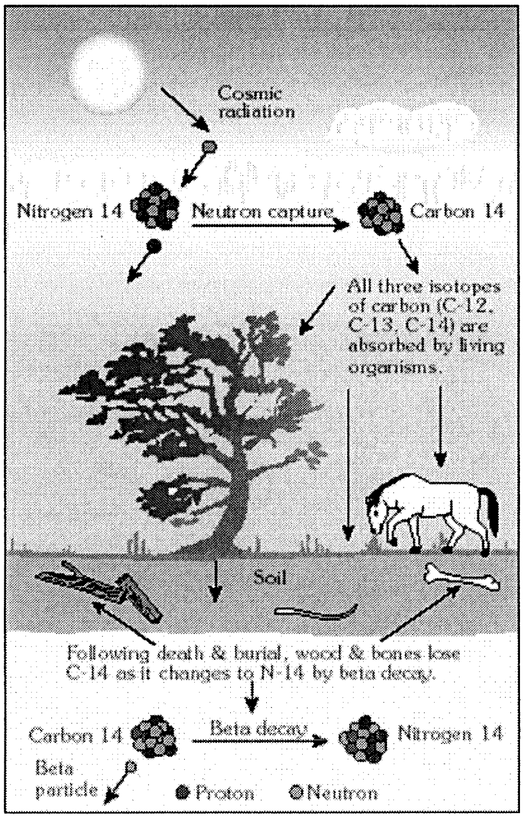
The number 'e'



Most elements come in several varieties called isotopes, which only differ in the number of neutrons that they contain. Most isotopes are unstable, and will decay into more stable isotopes or elements over time.

The decay time is measured by the time it takes half of the atoms to change into other forms and is called the half-life. A simple formula based on powers of the number 'e' connect the initial number of atoms to the remaining number after a time period has passed:

$$N(t)=a\ e^{-0.69t/T}$$

where T is the half-life in the same time units as t.

Problem 1 – What is the initial number of atoms at a time of t=0?
Answer: N(t) = a

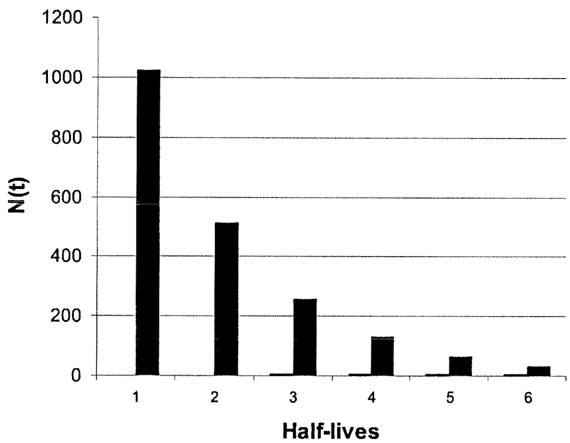
Problem 2 – Use a bar-graph to plot the function N(t) for a total of 6 half-lives with a = 2048.

Problem 3 - If a=1000 grams and T = 10 minutes, what will be the value of N(t) in when t = 1.5 hours?

Problem 4 – Carbon has an isotope called 'carbon-14' that decays to ordinary nitrogen in 5770 years. Suppose that a sample of plant material started out with 10 grams of carbon-14. If the half-life 5770 years, how many grams of carbon-14 will be present after 3,000 years?

Problem 1 – What is the initial number of atoms at a time of t=0?
Answer: **N(0) = a**

Problem 2 – Use a bar-graph to plot the function N(t) for a total of 6 half-lives with a = 2048. Answer: N = 1024, 512, 256, 128, 64, 32



Problem 3 - If a=1000 grams and T = 10 minutes, what will be the value of N(t) in when t = 1.5 hours?

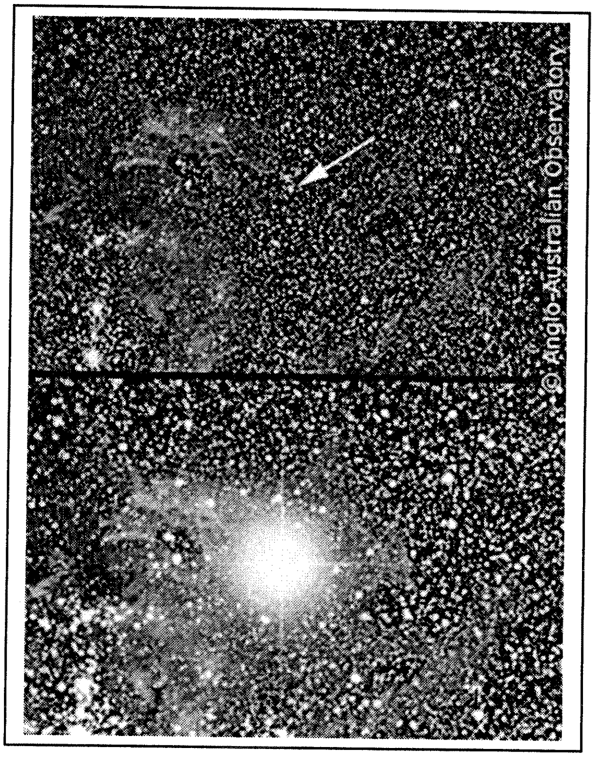
Answer: 1.5 hours = 90 minutes, so since t and b are now in the same time units:

N(90 minutes) = 1000 x $e^{(-0.69*90/10)}$
= 1000 x 0.002
= 2 grams

Problem 4 – Carbon has an isotope called 'carbon-14' that decays to ordinary nitrogen in 5770 years. Suppose that a sample of plant material started out with 10 grams of carbon-14. If the half-life is 5770 years, how many grams of carbon-14 will be present after 3,000 years?

Answer: a = 10 grams, T = 5770 years so N(t) = 10 $e^{-0.69(t/5770)}$
After t = 3000 years,
N(3000) = 10 $e^{-0.69(3000/5770)}$
N(3000) = 10 (0.7) **N(3000) = 7 grams.**

The number 'e'



After a star becomes a supernova, the light that its expanding gas produces fades over time. Astronomers have discovered that this fade-out is controlled by the light produced by the decay of radioactive nickel atoms.

This series of two photographs were taken by Dr. David Malin at the Anglo-Australian Observatory in 1987 and shows a before-and-after view of the supernova of 1987.

Careful studies of the brightness of this supernova in the years following the explosion reveal the 'radioactive decay' of its light.

Supernova 1987A produced 24,000 times the mass of our Earth in nickel-56 atoms, which were ejected into the surrounding space and began to decay to a stable isotope called cobalt-56. The half-life of nickel-56 is 6.4 days. Eventually the cobalt-56 atoms began to decay into stable iron-56 atoms. The half-life for the cobalt decay is 77 days.

Problem 1 – Using the half-life formula $N(t) = a\, e^{-0.69(t/T)}$, how much of the original nickel-56 (with T = 6.4 days) was still present in the supernova debris after 100 days?

Problem 2 – Assuming that no further light is produced by the nickel-56 decays after 100 days, and that for t > 100 days the light is produced by cobalt-56 decay:

A) Create a table showing the predicted brightness, L, of this supernova between 100 days and 900 days (2.5 years) after the explosion if at t=100 days the brightness of the supernova equals 80 million times that of the sun. (Answers to 2 significant figures;

B) Graph the data, called a light curve, L(t), for the first 500 days of the decay.

C) How long did it take for the supernova to fade until it exactly equaled the luminosity of our sun (L = 1.0)?

Problem 1 – Using the half-life formula $N(t) = a\, e^{-0.69(t/T)}$, how much of the original nickel-56 (with T = 6.4 days) was still present in the supernova debris after 100 days?

Answer: The paragraph says that the supernova produced 24,000 times the mass of the earth in nickel-56, so a = 24,000 and for T = 6.4 days we have

$N(100 \text{ days}) = 24000\, e^{-0.69(100/6.4)}$

$N(100 \text{ days}) = 24000\ (0.00021)$

$N(100 \text{ days}) =$ **0.5 times the Earth's mass!**

Problem 2 – Assuming that no further light is produced by the nickel-56 decays after 100 days, and that for t > 100 days the light is produced by cobalt-56 decay:

A) Create a table showing the predicted brightness, L, of this supernova between 100 days and 900 days (2.5 years) after the explosion if at t=100 days the brightness of the supernova equals 80 million times that of the sun (Answers to 2 significant figures); **Answer below.**

B) Graph the data, called a light curve, L(t), for the first 500 days of the decay. **Answer below.**

C) How long did it take for the supernova to fade until it exactly equaled the luminosity of our sun (L = 1.0)?

Answer: Solve $1.0 = 80 \text{ million}\ e^{(-0.69(t-100)/77)}$.

$\ln(1.0/80 \text{ million}) = -0.69(t-100)/77$ so $t - 100 = 77 \ln(80{,}000{,}000)/0.69$ and so **t = 2,131 days or 5.8 years.**

Days	L(t)
100	80,000,000
200	33,000,000
300	13,000,000
400	5,400,000
500	2,200,000
600	910,000
700	370,000
800	150,500
900	62,000